Small number of molecular ions in a linear trap can be sympathetically cooled with atomic ions and form a string crystal at the position, where the electric field is zero. Molecular ions in a string crystal are advantageous to measure the transition frequencies without Stark shift induced by the trap electric field, but it is required to localize small number of molecular ions in a single quantum state. $^{40}$CaH$^+$ molecular ion is advantageous to solve this problem, because (1) molecular ion with rotational constant of 141 GHz is localized in the vibrational-rotational ground state when the surrounding temperature is lower than 10 K, and (2) there is no hyperfine splitting in the $J = 0$ state. 

In this presentation, we propose to measure the $^{40}$CaH$^+$ $X^1\Sigma (v, N, F, M) = (0, 0, 1/2, \pm 1/2) \rightarrow (v_u, 0, 1/2, \pm 1/2) (v_u = 1, 2, 3, \ldots)$ transition with the uncertainty lower than $10^{-16}$. With these transitions, Zeeman shift is less than $10^{-16}$/G (given by the slight dependence of shielding effect by electron cloud on the vibrational state) and electric quadrupole shift is zero because of $F = 1/2$.

The $J = 0 \rightarrow 0$ transition is one-photon forbidden, and it can be observed also by Raman transition using two lasers. Stark shift induced by Raman lasers actually dominates the measurement uncertainty. When $v = 0 \rightarrow 1$ transition is observed using Raman lasers in the 6000-15000 /cm, Stark shift with saturation power is of the order of $1.5 \times 10^{-14}$ and it is higher for overtone transitions. With the following Raman laser frequencies, total Stark shift induced by two Raman lasers is zero.

- $v = 0 \rightarrow 1$ 24527 /cm and 23079 /cm
- $v = 0 \rightarrow 2$ 24600 /cm and 21745 /cm
- $v = 0 \rightarrow 3$ 26237 /cm and 22017 /cm
- $v = 0 \rightarrow 4$ 25354 /cm and 19814 /cm

The $^{40}$CaH$^+$ $X^1\Sigma (v, N, F, M) = (0, 0, 1/2, \pm 1/2) \rightarrow (v_u, 0, 1/2, \pm 1/2) (v_u = 1, 2, 3, \ldots)$ transition can be measured with the uncertainty lower than $10^{-16}$, and it is useful to test the variation in the proton-to-electron mass ratio.